Year 9

PLOTTING AND INTERPRETTING GRAPHS

Key Concept

Substitution – This is where you replace a number with a letter

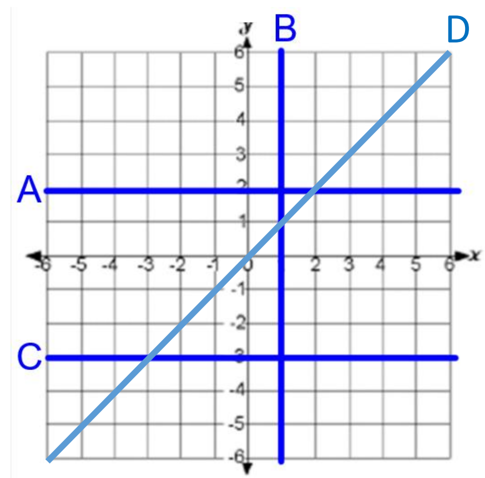
If a = 5 and b = 2

a + b =	5 + 2 = 7
a – b =	5 – 2 = 3
3a =	3 × 5 = 15
ab =	5 × 2 = 10
a^2 =	5^2 = 25

Key Words

Intercept: Where two graphs cross.
Gradient: This describes the steepness of the line.
y-intercept: Where the graph crosses the y-axis.
Linear: A linear graph is a straight line.
Quadratic: A quadratic graph is curved, u or n shape.

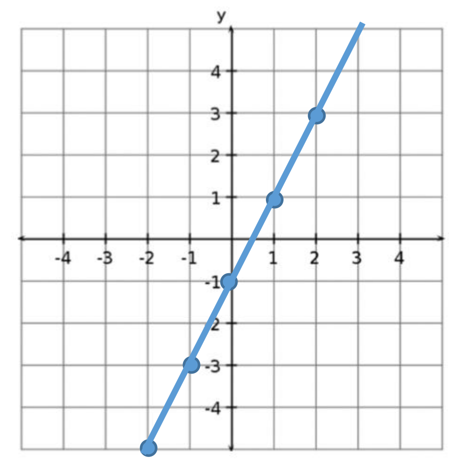
Examples



A: y = 2 B: x = 1
C: y = -3 D: y = x

Draw the graph of y = 2x - 1

X	- 2	- 1	0	1	2
Y	-5	-3	-1	1	3



Notice this graph has a gradient of 2 and a y-intercept of -1.

hegartymaths

Clip Numbers

206 - 210, 251

Tip

Parallel lines have the same gradient.

Formula

$$Gradient = \frac{difference\ in\ y's}{difference\ in\ x's}$$

Questions

1) What are the gradient and y-intercept of:

a) y = 4x – 3 b) y = 4 + 6x c) y = - 5x – 3

2) Draw the graph of y = 3x – 2 for x values from -3 to 3 using a table.

ANSWERS: 1) a) m = 4, c = -3 b) m = 6, c = 4 c) m = -5, c = -3

Year 9

STRAIGHT LINE GRAPHS AND EQUATION OF A LINE

Key Concepts

Coordinates in 2D are written as follows:

(x, y)

x is the value that is to the left/right

y is the value that is to up/down

Straight line graphs always have the equation:

$$y = mx + c$$

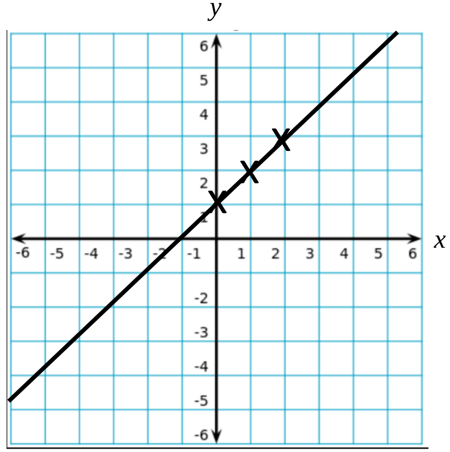
m is the **gradient** i.e. the steepness of the graph.

c is the **y intercept** i.e. where the graph cuts the y axis.

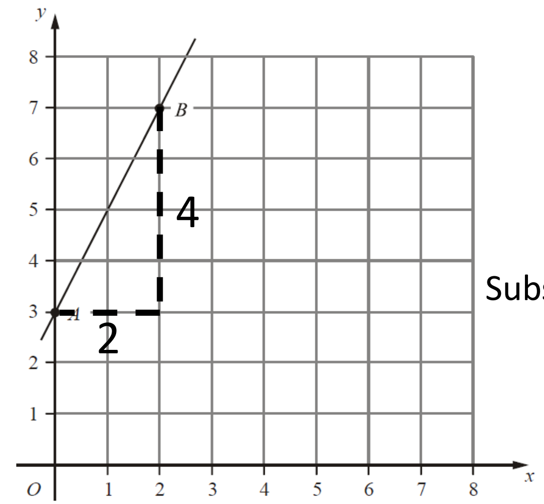
Examples

Plot the graph of $y = x + 1$

x	**0**	**1**	**2**
y	1	2	3



Calculate the equation of this line:



$$y = mx + c$$

$$m = \frac{4}{2}$$

$$= 2$$

$$y = 2x + c$$

Substitute in a coordinate: (2,7)

$$7 = (2 \times 2) + c$$

$$3 = c$$

$$y = 2x + 3$$

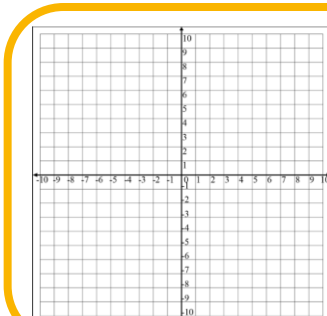
hegartymaths

199,200,205, 207-211,214

Key Words

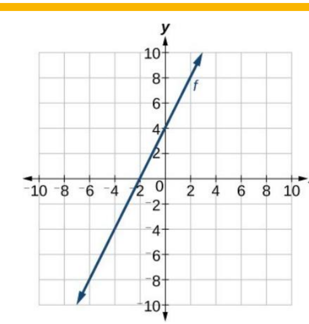
Coordinate

Gradient



1) Plot the line $y = 3x - 2$

2) Find the equation of the line for the attached graph.



ANSWERS: 2) $y = 2x + 4$

Year 9
SIMULTANEOUS EQUATIONS

Key Concepts

Simultaneous equations are when **more than one equation** are given, which involve **more than one variable**. The variables have the **same value** in each equation.

Examples

Two linear equations:

$$3x + 2y = 18$$
$$3x - y = 9 \quad \times 2$$

$$3x + 2y = 18$$
$$6x - 2y = 18 \quad +$$

SSS – Same Sign Subtract
DSA – Different Sign Add

$$9x = 36$$
$$x = 4$$

Substitute in $x = 4$ into an original equation

$$3x + 2y = 18$$
$$(3 \times 4) + 2y = 18$$
$$12 + 2y = 18$$
$$2y = 6$$
$$y = 3$$

One linear and one quadratic equation:

$$x^2 + y^2 = 17$$
$$y = x - 3$$

Substitute $y = x - 3$ into y in the quadratic equation.

$$x^2 + (x - 3)^2 = 17$$
$$x^2 + x^2 - 6x + 9 - 17 = 0$$
$$2x^2 - 6x - 8 = 0$$

Solve by factorising or using the quadratic formula.

$$x = 4 \text{ or } x = -1$$

Substitute the x values into the linear equation to find the corresponding y values.

$$When\ x = 4, \quad y = 4 - 3 = 1$$
$$When\ x = -1, \quad y = -1 - 3 = -4$$

hegartymaths

190-195

Key Words

Simultaneous
Substitution
Elimination
Linear
Quadratic

Solve each set of simultaneous equations:

1) $3x + 2y = 4$
   $4x + 5y = 17$

2) $x^2 + y^2 = 13$
   $x = y - 5$

ANSWERS: 1) $x = -2$ and $y = 5$ 2) $y = 3$ and $x = -2$, $y = 2$ and $x = -3$

Year 9
SOLVE SIMULTANEOUS EQUATIONS GRAPHICALLY

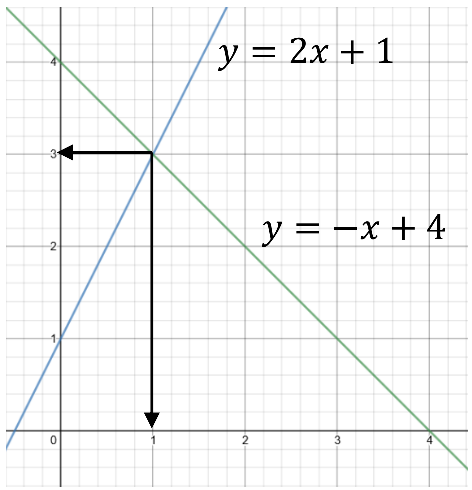
Key Concepts

Simultaneous equations are when **more than one equation** are given which involve **more than one variable**. The variables have the **same value** in each equation.

Simultaneous equations can be solved **graphically** whereby the **intersection** of the graphs gives the x and y values.

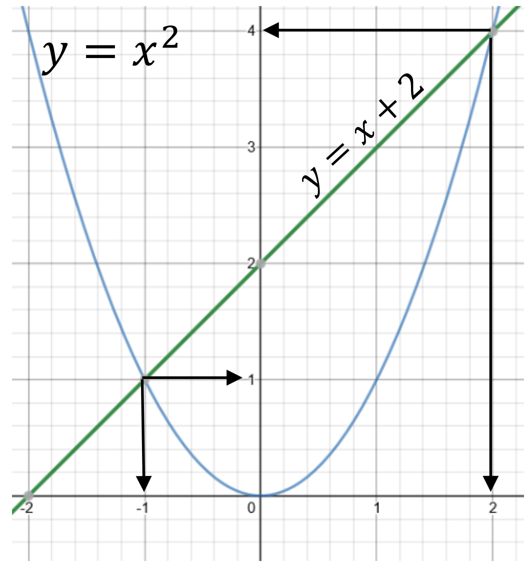
Examples

Solve graphically: $y = 2x + 1$
$y = -x + 4$



$x = 1$ and $y = 3$

Solve graphically: $y = x^2$
$y = x + 2$



$x = -1$ and $y = 1$

$x = 2$ and $y = 4$

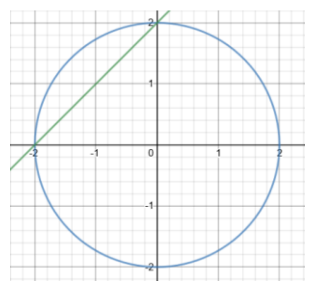
hegartymaths

259

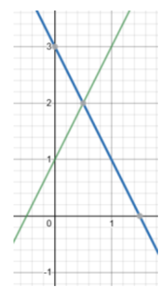
Key Words

Simultaneous
Equation
Intersection

1)



2)



Solve each set of simultaneous equations graphically.

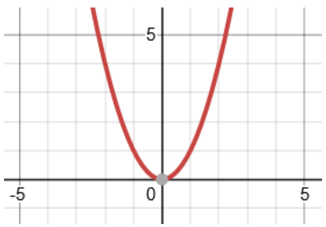
ANSWERS 1) $x = -2$ *and* $y = 0$, $x = 0$ *and* $y = 2$ 2) $x = 0.5$ *and* $y = 2$

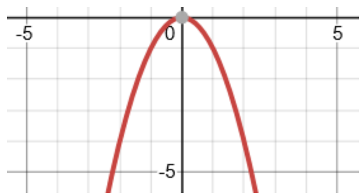
Year 9
QUADRATIC GRAPHS

Key Concepts

A quadratic graph will always be in the shape of a parabola.

$y = x^2$ $\qquad$ $y = -x^2$

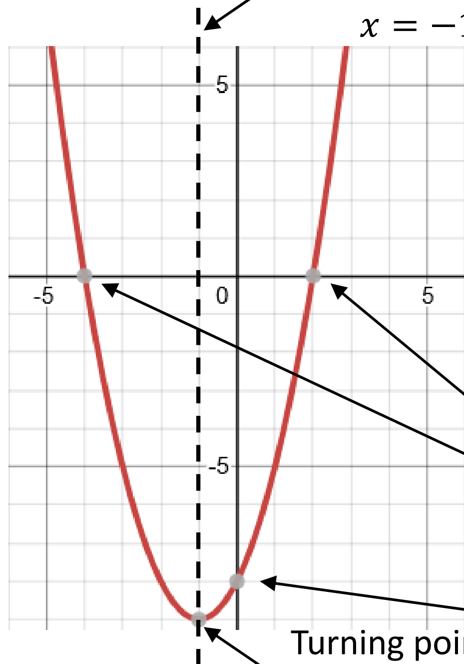




The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.

Examples

$$y = x^2 + 2x - 8$$



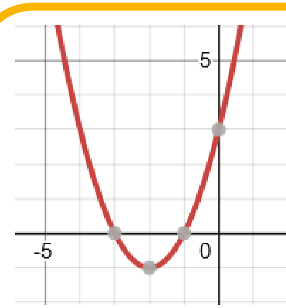
A quadratic equation can be solved from its graph.
The roots of the graph tell us the possible solutions for the equation.
There can be 1 root, 2 roots or no roots for a quadratic equation. This is dependant on how many times the graph crosses the x axis.

hegartymaths

251-256

Key Words

Quadratic
Roots
Intercept
Turning point
Line of symmetry



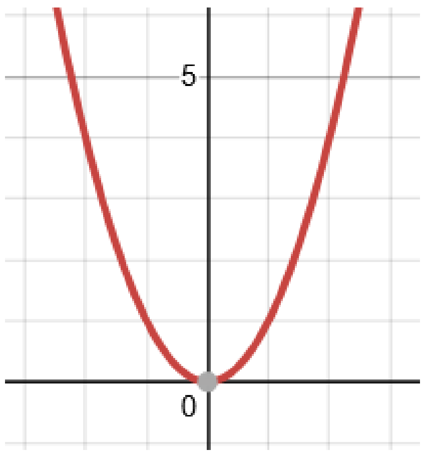
Identify from the graph of $y = x^2 + 4x + 3$:

1) The line of symmetry
2) The turning point
3) The y intercept
4) The two roots of the equation

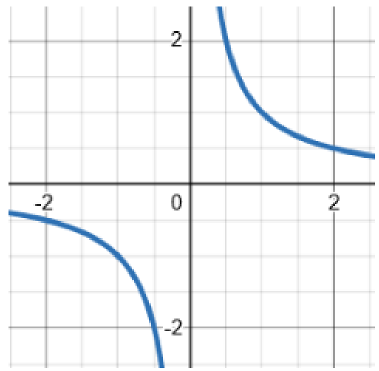
ANSWERS 1) $x = -2$ $\quad$ 2) $(-2, -1)$ $\quad$ 3) 3 $\quad$ 4) $x = -1$ *and* $x = -3$

Year 9
TYPES OF GRAPH

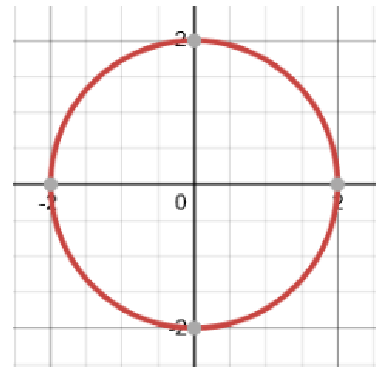
Examples



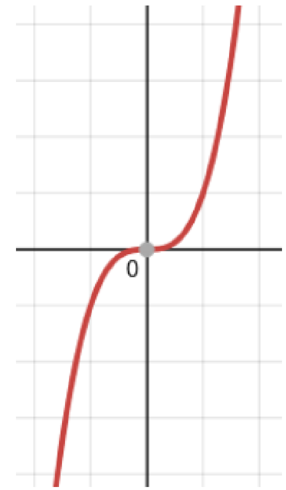
Quadratic graphs
$y = x^2$



Reciprocal graphs
$y = \frac{1}{x}$



Circle graphs
$x^2 + y^2 = 4$



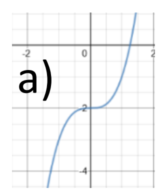
Cubic graphs
$y = x^3$

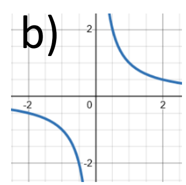
hegartymaths

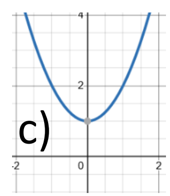
257, 298-301, 314

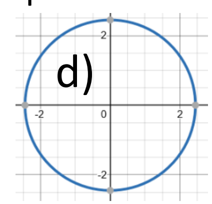
Key Words
Quadratic
Cubic
Reciprocal
Circle
Graph

Match the graph with the correct equation:

a)


b)


c)


d)


1) $x^2 + y^2 = 6$
2) $y = \frac{1}{x}$
3) $y = x^3 - 2$
4) $y = x^2 + 1$

ANSWERS: 1d 2b 3a 4c

Year 9
INTRODUCING PROBABILITY

Key Concept

Chance

Impossible — Unlikely — Even Chance — Likely — Certain

Probability

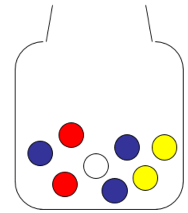
0	0.25	0.5	0.75	1
0%	25%	50%	75%	100%
0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1

Probabilities can be written as:
- Fractions
- Decimals
- Percentages

Key Words

Probability: The chance of something happening as a numerical value.
Impossible: The outcome cannot happen.
Certain: The outcome will definitely happen.
Even chance: The are two different outcomes each with the same chance of happening.
Expectation: The amount of times you expect an outcome to happen based on probability.

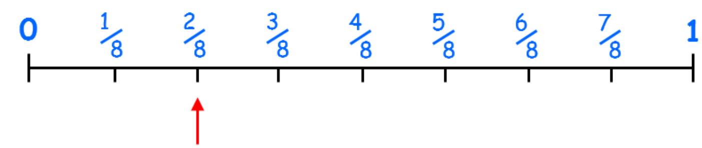
Examples



1) What is the probability that a bead chosen will be **yellow**.
Show the answer on a number line.

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

$$P(Yellow) = \frac{2}{8} = \frac{1}{4}$$



2) How many **yellow** beads would you **expect** if you pulled a bead out and replaced it 40 times?

$$\frac{1}{4} \times 40 = \frac{1}{4} of 40 = 10$$

hegartymaths

Clip Numbers
349 - 359

Tip
Probabilities always add up to 1.

Formula

$$Expectation = Probability \times no.of\ trials$$

Questions
In a bag of skittles there are 12 red, 9 yellow, 6 blue and 3 purple left.
Find: a) P(Red) b) P(Yellow) c) P(Red or purple) d) P(Green)

ANSWERS: 1) a) $\frac{12}{30} = \frac{2}{5}$ b) $\frac{9}{30} = \frac{3}{10}$ c) $\frac{15}{30} = \frac{1}{2}$ d) 0

Year 9
THEORETICAL PROBABILITY

Key Concepts

Probabilities can be described using **words** and **numerically**.

We can use **fractions, decimals or percentages** to represent a probability.

Theoretical probability is what should happen if all variables were fair.

All probabilities must **add to 1**.

The probability of something **NOT** happening equals:

$$1 - (probability\ of\ it\ happening)$$

Examples

Probability scale:

Impossible		Even chance		Certain
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{4}$
0	0.25	0.5	0.75	1
0%	25%	50%	75%	100%

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	3	5	2

1) What is the probability that a blue counter is chosen? $\frac{3}{19} = \frac{number\ of\ blue}{total\ number\ of\ counters}$

2) What is the probability that red is **not** chosen?

$$\frac{10}{19} = \frac{number\ of\ all\ other\ colours}{total\ number\ of\ counters}$$

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	$3x$	x-5	$2x$

A counter is chosen at random, the probability it is red is $\frac{9}{100}$. Work out the probability is black.

$$9 + 3x + x - 5 + 2x = 100$$
$$6x + 4 = 100$$
$$x = 16$$

Number of black counters = 16 – 5
= 11

Probability of choosing black = $\frac{11}{100}$

hegartymaths

349-353

Key Words

Theoretical
Probability
Fraction
Decimal
Percentage
Certain
Impossible
Even chance

	1	2	3
Prob	5	4	9

1a) Calculate the probability of choosing a 2.
b) Calculate the probability of not choosing a 3.

	1	2	3
Prob	0.37	$2x$	x

2) Calculate the probability of choosing a 2 or a 3.

ANSWERS: 1a) $\frac{4}{18}$ b) $\frac{9}{18}$ 2) P(2) = 0.42 P(3) = 0.21

Year 9
RELATIVE FREQUENCY

Key Concepts

Experimental probability differs to theoretical probability in that it is based upon the **outcomes from experiments**. It may not reflect the outcomes we expect.

Experimental probability is also known as the **relative frequency** of an event occurring.

Estimating the number of times an event will occur:

Probability × no. of trials

Examples

Colour	red	blue	white	black
Prob	x	0.2	0.3	x

A spinner is spun, it has four colours on it.
The relative frequencies of each colour are recorded.
The relative frequency of red and black are the same.

a) What is the relative frequency of red?

$$1-(0.2+0.3)=0.5$$
$$x=\frac{0.5}{2}=0.25$$

b) If the spinner is spun 300 times, how many times do you expect it to land on white?

$$0.3\times 300=90$$

hegartymaths

355-357

Key Words

Experimental
Relative frequency
Fraction
Decimal
Probability
Estimate

Number	1	2	3	4
Prob	x	0.46	0.28	x

A spinner is spun which has 1,2,3,4 on it. The probability that a 1 and a 4 are spun are equal.

a) What is the probability that a 4 is landed on?

b) If the spinner is spun 500 times how many times do we expect it to land on a 2?

ANSWERS: a) 0.13 b) 230

Year 9
LISTING OUTCOMES AND SAMPLE SPACE

Key Concepts

When there are a number of different possible outcomes in a situation we need a **logical** and **systematic** way in which to view them all.

We can be asked to **list** all possible outcomes e.g. choices from a menu, order in which people finish a race.

We can also use a **sample space diagram**. This records the possible outcomes of two different events happening.

Examples

Starter	Main
Fishcake Melon	Lasagne Beef Salmon

List all of the combinations possible when one starter and one main are chosen.

F, L   M, L
F, B   M, B
F, S   M, S

Note: You can write the initials of each option in a test. You do not need to write out the full word.

Two dice are thrown and the possible outcomes are shown in the sample space diagram below:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

1) What is the probability that 2 numbers which are the same are rolled?

$$\frac{6}{36} = \frac{outcomes\ where\ numbers\ are\ the\ same}{total\ number\ of\ outcomes}$$

2) What is the probability that two even numbers are rolled?

$$\frac{9}{36} = \frac{outcomes\ where\ numbers\ are\ both\ even}{total\ number\ of\ outcomes}$$

hegartymaths

358-359, 370-371

Key Words

List
Outcome
Sample space
Probability

1) Abe, Ben and Carl have a race. List all of the options for the order that the boys can end the race.

		Spinner		
Coin		Red	Green	Blue
	Heads	H,R	H,G	H,B
	Tails	T,R	T,G	T,B

2a) What is the probability that a head is landed on?
b) What is the probability that a head and a green are landed on?

ANSWERS: 1) ABC, ACB, BAC, BCA, CAB, CBA 2a) $\frac{3}{6}$ b) $\frac{1}{6}$

Year 9
TWO WAY TABLES AND PROBABILITY TABLES

Key Concepts

Two way tables are used to tabulate a number of pieces of information.

Probabilities can be formulated easily from two way tables.

Probabilities can be written as a **fraction, decimal or a percentage** however we often work with fractions. You do not need to simplify your fractions in probabilities.

Estimating the number of times an event will occur

Probability × no. of trials

Examples

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	$3x$	x-5	$2x$

A counter is chosen at random, the probability it is red is $\frac{9}{100}$. Work out the probability is black.

$$9 + 3x + x - 5 + 2x = 100$$
$$6x + 4 = 100$$
$$x = 16$$

Number of black counters = 16 – 5

= 11

Probability of choosing black = $\frac{11}{100}$

80 children went on a school trip. They went to London or to York.

23 boys and 19 girls went to London. 14 boys went to York.

	London	York	Total
Girls	19	**24**	**43**
Boys	23	14	**37**
Total	**42**	**38**	80

What is the probability that a person is chosen that went to London? $\frac{42}{80}$

If a girl is chosen, what is the probability that she went to York? $\frac{24}{38}$

hegartymaths

353,

422-424

Key Words

Two way table

Probability

Fraction

Outcomes

Frequency

	1	2	3
Prob	0.37	$2x$	x

1a) Calculate the probability of choosing a 2 or a 3.

b) Estimate the number of times a 2 will be chosen if the experiment is repeated 300 times.

2a) Complete the two way table:

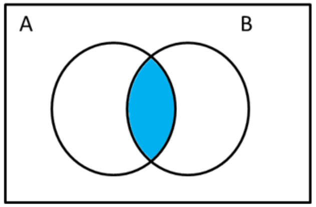
	Year Group			Total
	9	10	11	
Boys			125	407
Girls		123		
Total	303	256		831

b) What is the probability that a Y10 is chosen, given that they are a girl .

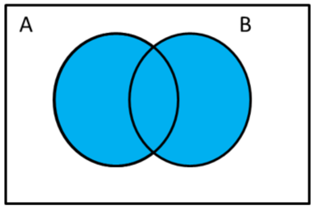
ANSWERS: 1) a) P(2) = 0.42 P(3) = 0.21 b) 0.42×300=126   2b) 123/424

Year 9
FURTHER PROBABILITY

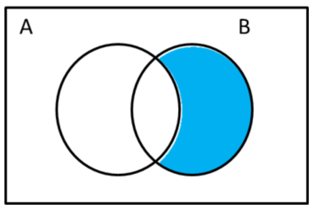
Key Concept



$P(A \cap B)$



$P(A \cup B)$



$P(A' \cap B)$

hegartymaths

359,360, 374-388, 422-424

Key Words

Probability: The chance of something happening as a numerical value.

Impossible: The outcome cannot happen.

Certain: The outcome will definitely happen.

Even chance: The are two different outcomes each with the same chance of happening.

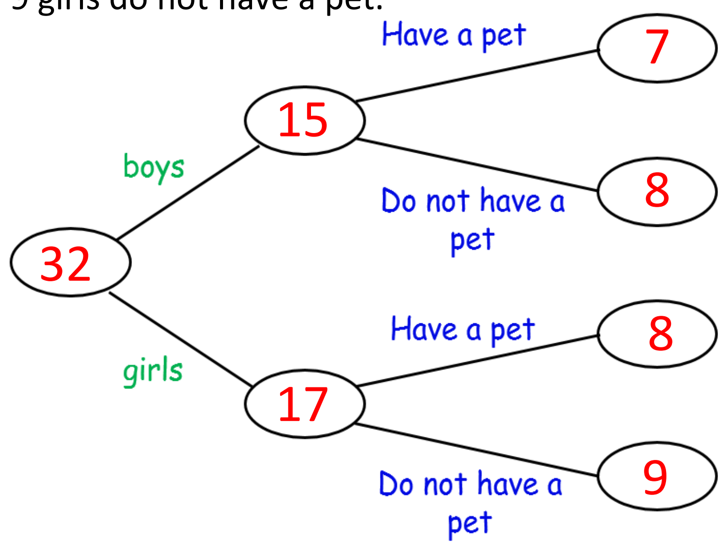
Mutually Exclusive: Two events that cannot both occur at the same time.

Formula

$$P(A \cap B) = P(A) \times P(B)$$
$$P(A \cup B) = P(A) + P(B)$$
$$or \quad (non\ ME) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Examples

In Hannah's class there are 32 students.
15 of these students are boys.
7 of the boys have a pet.
9 girls do not have a pet.



$$P(boy) = \frac{15}{32}$$

$$P(Girl\ with\ pet) = \frac{8}{32}$$

Questions

1) Draw a two-way table for the question above.
2) Find the probability that a pupil chosen is a boy with no pets.
3) A girl is chosen, what is the probability she has a pet?

ANSWERS: 2) $\frac{8}{32}$ 3) $\frac{8}{17}$

Year 9
PROBABILITY TREE DIAGRAMS

Key Concepts

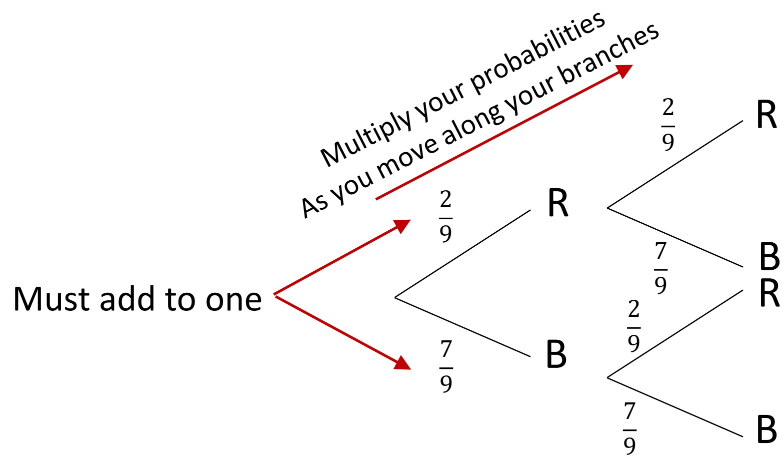
Independent events are events which do not affect one another.

Dependent events affect one another's probabilities. This is also known as **conditional probability**.

We **multiply** two probabilities when one event follows another.

Examples

There are red and blue counters in a bag.
The probability that a red counter is chosen is $\frac{2}{9}$.
A counter is chosen and **replaced**, then a second counter is chosen.
Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



Prob of two reds:

$$\frac{2}{9} \times \frac{2}{9} = \frac{4}{81}$$

Prob of two blues :

$$\frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$$

Prob of same colours:

$$\frac{4}{81} + \frac{49}{81} = \frac{53}{81}$$

hegartymaths

361-362, 364, 368-369

Key Words

Independent
Dependant
Conditional
Probability
Fraction
Multiply

There are blue and green pens in a drawer.
There are 4 blues and 7 greens.
A pen is chosen and then **replaced**, then a second pen is chosen.
Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

ANSWERS: 56/121

Year 9
VENN DIAGRAMS

Key Concepts

Venn diagrams show all possible relationships between different sets of data.

Probabilities can be derived from Venn diagrams. Specific notation is used for this:

$P(A \cap B)$ = Probability of A **and** B

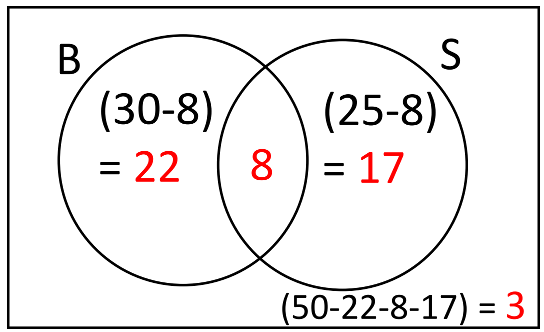
$P(A \cup B)$ = Probability of A **or** B

$P(A')$ = Probability of **not** A

Example

Out of 50 people surveyed:

30 have a brother
25 have a sister
8 have both a brother and sister



a) Complete the Venn diagram

b) Calculate:

i) $P(A \cap B)$ $= \frac{8}{50}$ ii) $P(A \cup B)$ $= \frac{47}{50}$ iii) $P(B')$ $= \frac{20}{50}$

iv) The probability that a person with a sister, does not have a brother.
$= \frac{8}{25}$

hegartymaths

372-388, 391

Key Words
Venn diagram
Union
Intersection
Probability
Outcomes

40 students were surveyed:

20 have visited France
15 have visited Spain
10 have visited both France and Spain

a) Complete a Venn diagram to represent this information.

b) Calculate:

i) $P(F \cap S)$ ii) $P(F \cup S)$ iii) $P(S')$

iv) The probability someone who has visited France, has not gone to Spain.

ANSWERS: bi) 10/40 ii) 25/40 iii) 25/40 iv) 20/40 = 1/2